

# Dynamical Restoration of $Z_N$ Symmetry in $SU(N)$ +Higgs Theories

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## Abstract

We study the  $Z_N$  symmetry in  $SU(N)$ +Higgs theories with the Higgs field in the fundamental representation. The distributions of the Polyakov loop show that the  $Z_N$  symmetry is explicitly broken in the Higgs phase. On the other hand inside the Higgs symmetric phase the Polyakov loop distributions and other physical observables exhibit the  $Z_N$  symmetry. This effective realization of the  $Z_N$  symmetry in the theory changes the nature of the confinement-deconfinement transition. We argue that the  $Z_N$  symmetry will lead to time independent topological defect solutions in the Higgs symmetric deconfined phase which will play important role at high temperatures.

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## I. INTRODUCTION

It is well known that most phenomena in pure  $SU(N)$  gauge theories do not depend on the representations of the gauge fields [1–9]. It is considered that both the fundamental and adjoint representations are equally valid representations of the non-abelian gauge fields and differences specific to representations are in general considered unphysical. The preference to a particular representation arises when the gauge fields are coupled to the matter fields. In the presence of the matter fields the two representations of the gauge fields are not equivalent. In quantum field theories such as the quantum chromodynamics ( $QCD$ ) and the electroweak ( $EW$ ) theory, which describe the strong and electro-weak forces of nature respectively, the matter fields are in the fundamental representations. The gauge invariance of these theories requires that the gauge fields also be in the fundamental representation. Given that there is a clear preference to the fundamental representation of the gauge fields, the physics aspects specific to this representation can play important role in these theories.

One of the important physics issue which arises in the fundamental representation is the  $Z_N$  symmetry. At finite temperatures the gauge fields are periodic along the temporal direction [10]. This boundary condition requires that in the temporal direction the gauge transformations are periodic up to a factor  $z$ , which is an element of the center ( $Z_N$ ) of the gauge group  $SU(N)$ . A gauge transformation which is periodic upto a phase factor  $z$  (in the temporal direction) non-trivially transforms the Polyakov loop ( $L$ ), which is the trace of a path ordered product of exponentials of the temporal gauge field  $A_0$  along the shortest temporal loop. The Polyakov loop picks up the element  $z$  as a phase factor, i.e  $L \rightarrow zL$  [10]. All possible gauge transformations of the Polyakov loop then form the  $Z_N$  symmetry group. This symmetry plays an important role in the finite temperature confinement-deconfinement transition in pure  $SU(N)$  gauge theories. In the deconfined phase the Polyakov loop acquires a non-zero expectation value which leads to the spontaneous breaking of the  $Z_N$  symmetry. On the other hand in the confined phase it has zero expectation value. This property of the Polyakov loop across the confinement-deconfinement transition makes it an ideal candidate for an order parameter for this transition[11].

Even though the above non-periodic gauge transformations preserve the boundary conditions of the gauge fields they do not preserve the temporal boundary condition of the matter fields in the fundamental representation. After a gauge transformation for which  $z \neq I$  ( $I$  is the identity element of  $Z_N$ ) bosonic(fermionic) matter fields are no more periodic(anti-periodic). These gauge transformations therefore can not act on the matter fields. However it still makes sense to consider

these  $Z_N$  gauge transformations by restricting their actions only to the gauge fields. These transformations, which are not like the conventional gauge transformations acting both on the gauge and the matter fields, will not leave the action of the full theory invariant. However a given gauge field configuration as well as its  $Z_N$  transformations are both valid configurations and will contribute to the partition function of the full theory. Their individual contribution to the partition function will decide the relative “Boltzmann” probability of these two configurations in a thermal ensemble. Even though the classical action does not have the  $Z_N$  symmetry ultimately the fluctuations of the fields will decide if the  $Z_N$  symmetry is relevant in presence of matter fields. Here by  $Z_N$  symmetry we imply that the gauge transformations are acting only on the gauge fields. The Higgs fields can be gauge transformed only when the gauge transformations correspond to the identity of  $Z_N$ .

The issue of  $Z_N$  symmetry in the presence of fundamental matter fields has been extensively studied in the literature [12–16]. It was shown that the 1-loop perturbative effective potential for the Polyakov loop has meta-stable states with negative entropy [17] in the presence of fermions. In these studies, however, only the zero mode of the Polyakov loop is coupled to the matter fields. Higher modes of the Polyakov loop, which actually give rise to the spontaneous breaking of the  $Z_N$  symmetry, may resolve the problem of negative entropy. Subsequent studies using effective models [18, 19] and lattice QCD studies [20, 21] have shown that the presence of fermions acts as an external effective field on the Polyakov loop thereby breaking the  $Z_N$  symmetry explicitly. Although there have been a lot of non-perturbative studies on the confinement-deconfinement transition of  $SU(N)$  gauge theories coupled to fundamental bosonic fields [22, 25, 26] but very few have addressed the issue of the  $Z_N$  symmetry in these theories. In this work we carry out non-perturbative study of the  $Z_N$  symmetry in the presence of bosonic matter fields in the fundamental representation. More efforts are needed to address the issues related to the  $Z_N$  symmetry in the presence of matter fields such as the thermodynamic properties of meta-stable states, strength of the symmetry breaking field etc. through higher order corrections to the effective potential and by non-perturbative Monte Carlo simulations.

To study the  $Z_N$  symmetry we focus mainly on the properties of the Polyakov loop as it is most sensitive to this symmetry. We compute the distribution of the Polyakov loop using the Monte Carlo simulations of the partition function. We have carried out simulations for the cases of  $N = 2$  and  $N = 3$ . The distribution of the Polyakov loop is found to be similar to the distribution of the magnetization in the  $N$ -state Potts model (which has  $Z_N$  symmetry) in the presence of the external field. The external field causes asymmetry in the distributions of the magnetization which otherwise has the  $Z_N$  symmetry. The larger the external field is larger is the asymmetry

in the distribution of the magnetization. In the present case the asymmetry of the Polyakov loop distribution is found to vary with the Higgs condensate. It is observed that the distribution has large(small) asymmetry when the condensate is large(small). These results suggest that the external field for the Polyakov loop (the  $Z_N$  symmetry) depends on the Higgs field. It is never expected that the external field vanish as long as there is interaction between the gauge and the Higgs fields. Surprisingly it is found that for a suitable choice of external parameters, when the system is in the Higgs symmetric phase, the Polyakov loop distribution exhibits the  $Z_N$  symmetry. The simulation results also show that the different  $Z_N$  states in the deconfined phase have the same free energy. This implies the vanishing of the effective external field. This occurs while there is non-zero interaction (correlation) between the gauge and the Higgs fields. In this case the nature of the confinement-deconfinement transition is almost same as in the pure gauge case. Apart from affecting the confinement-deconfinement transition the  $Z_N$  restoration in the theory will lead to presence of domain walls and strings defects ( $N > 2$ ) at very high temperatures in the deconfined phase. Previously the effective potential calculations have shown that the  $Z_N$  symmetry is restored only in the limit of infinitely heavy Higgs mass, that is basically when the Higgs field decouples from the gauge fields. In contrast in our non-perturbative studies the  $Z_N$  symmetry is realized even when the Higgs has finite mass and its interaction with the gauge fields is non-zero. It would be interesting to investigate this symmetry in the presence of fundamental fermion fields in view of its restoration in the presence of the Higgs field. We mention here that conventionally symmetry restoration means that the distribution of the order parameter (the Polyakov loop in the present context) is symmetrically peaked around zero. In the present context by symmetry restoration we imply that the full theory exhibits the corresponding symmetry.

The paper is organized as follows. In the following in section-II we discuss the  $Z_N$  symmetry in  $SU(N)$ +Higgs theories. In section-III we present our numerical simulations and results. In section-IV we present our discussions and conclusions.

## II. THE $Z_N$ SYMMETRY IN THE PRESENCE OF FUNDAMENTAL HIGGS FIELDS

The Euclidean  $SU(N)$  action for the gauge fields  $A_\mu^a$  ( $a = 1, 2, \dots, N^2 - 1$ ) in the fundamental representation is given by,

$$S = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} \text{Tr} (F^{\mu\nu} F_{\mu\nu}) \right\}. \quad (1)$$

$V$  is spatial volume and  $\beta$  is the extent in temporal direction. The gauge field strength  $F_{\mu\nu}$  is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu], \quad A_\mu = A_\mu^a T^a. \quad (2)$$

The  $N \times N$  matrices  $T^a$ 's are the generators of the  $SU(N)$  gauge group.  $g$  is the gauge coupling constant. In the Euclidean theory the gauge fields  $A_\mu^a$  are periodic in the temporal direction, i.e  $A_\mu^a(\vec{x}, 0) = A_\mu^a(\vec{x}, \beta)$ . Under a gauge transformation  $U(\vec{x}, \tau) \in SU(N)$  the gauge fields transform as

$$A_\mu \longrightarrow U A_\mu U^{-1} + \frac{1}{g} (\partial_\mu U) U^{-1}. \quad (3)$$

Though the gauge fields must be periodic the gauge transformations  $U(\vec{x}, \tau)$  need not be periodic in the temporal direction. The invariance of the pure gauge action and the periodicity of the gauge fields both can be satisfied by gauge transformations which are periodic up to a factor  $z$  such as,

$$U(\vec{x}, \tau = 0) = z U(\vec{x}, \tau = \beta). \quad (4)$$

Where  $z \in Z_N$  and  $Z_N$  is the center of the gauge group  $SU(N)$  [11, 27]. The Polyakov loop ( $L$ ) which is the path ordered product of links in the temporal direction,

$$L(\vec{x}) = \frac{1}{N} \text{Tr} \left\{ P e^{(-ig \int_0^\beta A_0 d\tau)} \right\} \quad (5)$$

transforms as  $L \longrightarrow zL$  under a gauge transformation (Eq.(3)) with the boundary condition Eq.(4). Consequently the Polyakov loop behaves like a  $Z_N$  spin and plays the role of an order parameter for the pure gauge confinement-deconfinement transition. Note that  $L$  is the trace of an  $SU(N)$  matrix. For  $N = 2$  the range of values  $L$  can take is  $[-1, 1]$ . For  $N > 2$  it can take any value in a  $n$ -polygon in the complex plane whose vertices are given by  $e^{i\frac{2\pi n}{N}}, n = 0, 1, N-1$ .

The modified action which describes the interaction of the gauge fields and the Higgs field  $\Phi$  is given by,

$$S = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} \text{Tr} (F^{\mu\nu} F_{\mu\nu}) + \frac{1}{2} |D_\mu \Phi|^2 + \frac{m^2}{2} \Phi^\dagger \Phi + \frac{\bar{\lambda}}{4!} (\Phi^\dagger \Phi)^2 \right\}. \quad (6)$$

The  $\Phi$  field is a  $N \times 1$  column matrix with complex elements.  $m, \bar{\lambda}$  are the bare mass and the self-interaction strength of the  $\Phi$  field respectively. The covariant derivative  $D_\mu \Phi$  is defined as

$D_\mu \Phi = \partial_\mu \Phi + ig A_\mu \Phi$ . Being a bosonic field,  $\Phi$  satisfies periodic boundary condition in the temporal direction, i.e  $\Phi(\vec{x}, 0) = \Phi(\vec{x}, \beta)$ . Under a gauge transformation  $U(\vec{x}, \tau)$  the  $\Phi$  field transforms as,

$$\Phi' = U\Phi. \quad (7)$$

It is obvious that  $\Phi'$  is periodic only when the gauge transformations are periodic. Therefore the gauge transformations which are not periodic are not allowed to act on the matter fields. Thus  $Z_N$  group is not a symmetry of the classical action (Eq.6). However the actual manifestation of the  $Z_N$  symmetry can be seen only after the fluctuations of the gauge and matter fields are included as fluctuations play dominant role in these theories. The change in the action due  $Z_N$  transformation acting only on the gauge fields can be compensated by fluctuations of the Higgs field. This leads to the complete realization/restoration of the  $Z_N$  symmetry. In the following we describe the numerical Monte Carlo simulations and results.

### III. SIMULATIONS OF THE $SU(N)$ +HIGGS MODEL

In the Monte Carlo simulations of  $SU(N) + Higgs$  model, the 4-dimensional Euclidean space is replaced by a discrete lattice. The lattice sites are represented by  $n = (n_1, n_2, n_3, n_4)$  where  $n_i$ 's are integers. The gauge field  $A_\mu$  is replaced by the link variables  $U_\mu = \exp(-iagA_\mu)$ , where  $a$  is the lattice constant/spacing. The link variable  $U_\mu(n)$  lives on the link between the sites  $n$  and  $n + \hat{\mu}a$ , where  $\hat{\mu}$  is a unit vector in the  $\mu$ th direction. The Higgs field  $\Phi(n)$  lives on the lattice site  $n$ . The discretized lattice action is given by,

$$S = \beta \sum_p \frac{1}{2} \text{Tr}(2 - U_p - U_p^\dagger) - \kappa \sum_\mu \text{Re} \left[ (\Phi_{n+\mu}^\dagger U_{n,\mu} \Phi_n) \right] + \frac{1}{2} (\Phi_n^\dagger \Phi_n) + \lambda \left( \frac{1}{2} (\Phi_n^\dagger \Phi_n) - 1 \right)^2 \quad (8)$$

where  $U_p$  is the product of links in an elementary square  $p$  on the lattice. The  $\Phi$  field and other parameters are all dimensionless in the discretized action [28]. The Polyakov loop  $L(n_i)$  at a spatial site  $n_i$  is trace of the path ordered product of all temporal link variables on the temporal loop going through  $n_i$ . A  $Z_N$  rotation can be carried out by multiplying all temporal links on a fixed temporal slice of the lattice by an element of the  $Z_N$  group. This operation leaves all terms of the above action invariant except the  $\kappa$  dependent term. This term is solely responsible for the explicit breaking of the  $Z_N$  symmetry.

In the simulations an initial configuration of  $\Phi_n$  and  $U_{\mu,n}$  is selected. This initial configuration

is then repeatedly updated to generate a Monte Carlo history. In an update a new configuration is generated from an old one according to the Boltzmann probability factor  $e^{-S}$  and the principle of detailed balance. These conditions are implemented using pseudo heat-bath algorithm for the  $\Phi$  field [29] and the standard heat-bath algorithm for the link variables  $U_\mu$ 's [30, 31]. Apart from updating procedure over relaxation methods are also used to reduce the autocorrelations between adjacent configurations along the Monte Carlo trajectory [32].

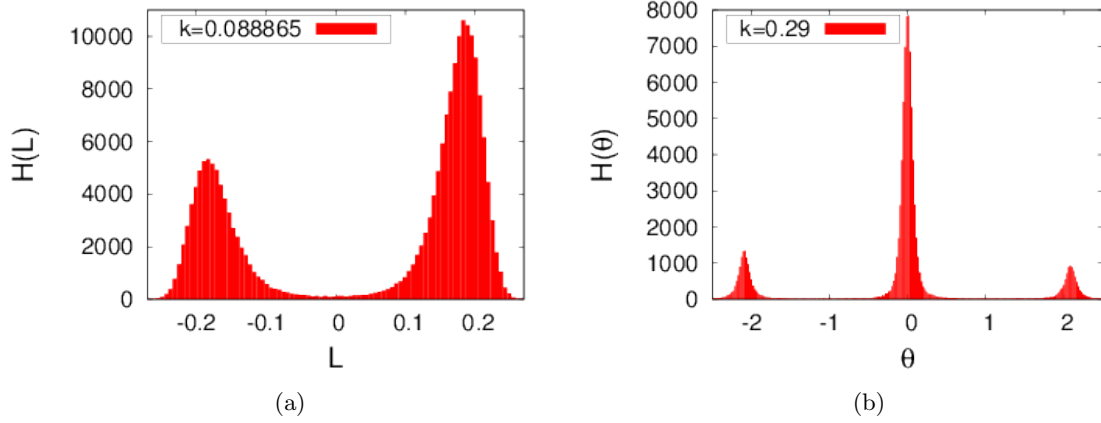


FIG. 1: Distribution of Polyakov loop in the Higgs broken phase for (a)  $SU(2)$ ,  $16^3 \times 4$  lattice and (b)  $SU(3)$ ,  $8^3 \times 4$  lattice.

The simulations are carried out for different values of  $\beta, \kappa$  and  $\lambda$ . The coupling  $\lambda$  controls the nature of the Higgs transition. The transition is first order(crossover) for small(large) values of  $\lambda$ . For a fixed  $(\lambda, \beta)$  the parameter  $\kappa$  plays the role of the transition parameter for the Higgs transition. For high  $\kappa (\kappa > \kappa_c)$  the system is found to be in the Higgs phase with a non-zero Higgs condensate. With decrease in  $\kappa$  the condensate starts to melt and at the critical point  $\kappa = \kappa_c$  the system undergoes transition to the Higgs symmetric phase. For  $\kappa < \kappa_c$  the Higgs condensate vanishes. For our purpose it suffices to fix the coupling  $\lambda$  and study the  $Z_N$  symmetry at various values of  $\kappa$ . Given a  $(\lambda, \kappa)$  small(large)  $\beta$  corresponds to the confinement(deconfinement) phase. The confinement-deconfinement transition takes place at the critical point  $\beta = \beta_c$  [22–26]. To study the  $Z_N$  symmetry at different  $\kappa$  we compute the Polyakov loop distribution and simulate confinement-deconfinement transition. We also compute various observables which are sensitive to the  $Z_N$  symmetry. In Fig.1a we show the Polyakov loop distribution( $H(L)$ ) in the deconfined phase for  $N = 2$  for  $\lambda = 0.005$  and  $\kappa = 0.088865$ . The explicit breaking of  $Z_2$  symmetry is clearly seen in the distribution  $H(L)$ . The local maximum here corresponds to the meta-stable state of the system. For  $N \geq 3$  the Polyakov loop is complex. For better illustration we show the distribution

of phase of the Polyakov loop  $H(\theta)$  instead of  $H(L)$  on the complex plain. In Fig.1b we show  $H(\theta)$  for  $\lambda = 0.1$  and  $\kappa = 0.29$  for  $N = 3$ . The peak at  $\theta = 0$  clearly dominates the other two local maxima as a result of the  $Z_3$  explicit symmetry breaking. It has been observed that the asymmetry in the above distributions increases when  $\kappa$  is increased further. Beyond some value of  $\kappa$  (which depends on  $\lambda$  and  $N$ ) the local maxima(the meta-stable states) disappear.

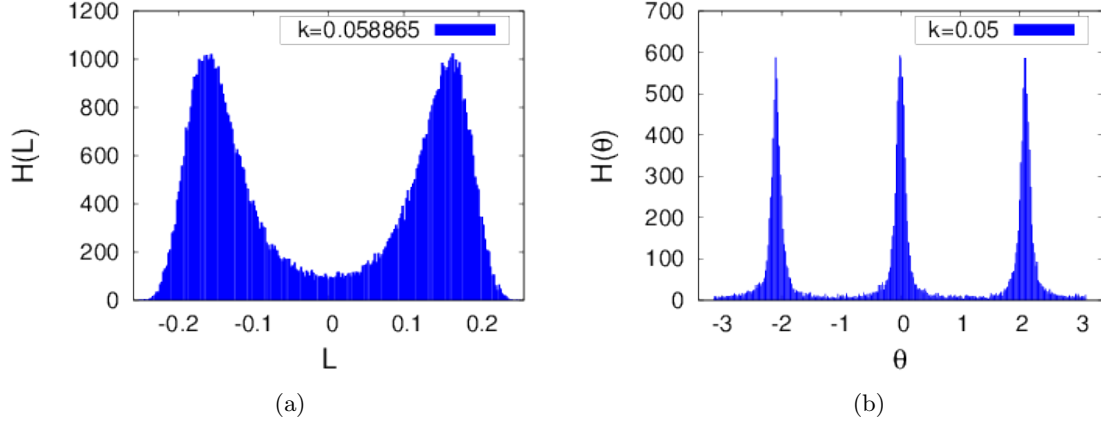


FIG. 2: Distribution of Polyakov loop in the Higgs symmetric phase for (a)  $SU(2)$ ,  $16^3 \times 4$  lattice and (b)  $SU(3)$ ,  $8^3 \times 4$  lattice.

The  $Z_N$  symmetry is supposed to be there only when  $\kappa = 0$  as the matter and gauge fields decouple. Surprisingly it is found in our simulations that in the Higgs symmetric phase ( $0 < \kappa < \kappa_c$ ) the distributions of the Polyakov loop exhibit the  $Z_N$  symmetry. This is evident in the distribution ( $H(L)$ ) of the Polyakov loop for  $N = 2$  shown in Fig.2a. Similarly the distribution  $H(\theta)$  for  $N = 3$  shows the  $Z_3$  symmetry. For small  $\kappa$  the Higgs correlation length can become shorter than the lattice spacing, i.e  $\Phi_n$  and  $\Phi_{n+\mu}$  are not correlated. With the product  $\Phi_n \Phi_{n+\mu}^\dagger$  having no preferential orientation with respect to  $U_\mu(n)$  the  $\kappa$  term in Eq.(8) can not affect the  $Z_N$  symmetry. Though this is plausible but our simulations suggest that this is not the reason for the  $Z_N$  realization/restoration. The  $\kappa$  term was found to be non-zero finite. The product  $\Phi_n \Phi_{n+\mu}^\dagger$  tend to align with  $U_\mu(n)$ . When a  $Z_N$  rotation ( $(\Phi, U) \rightarrow (\Phi, U_g)$ ) is carried out on any configuration from the thermal ensemble the resulting configuration is found to be out of equilibrium. This is because the new configuration has far higher action (Eq.(8)) than any configuration in the thermal ensemble. Interestingly this cost in the action can be compensated by varying the  $\Phi$  field, i.e  $\Phi \rightarrow \Phi'$ , coupled with the gauge rotation of the links.  $\Phi'$  can be obtained by Monte Carlo updates of  $\Phi$ , though it is not clear how  $\Phi$  and  $\Phi'$  are related. We observed that the symmetry  $(\Phi, U) \rightarrow (\Phi', U_g)$  is there only in the Higgs symmetric phase ( $\kappa < \kappa_c$ ) and when the number of



lattice points in the temporal direction is  $N_\tau \geq 4$ .

To see the  $Z_N$  symmetry in the Polyakov loop distribution, the tunneling between the different  $Z_N$  sectors has to be high. The tunneling rate decreases away from the transition point and also for larger lattice size. For these cases even for a reasonably large statistics it is unlikely that the population of the different Polyakov loop sectors will be found same. For example, for  $\beta = 2.38$  and  $16^3 \times 4$  lattice we do not see any tunneling between the different  $Z_2$  sectors up to  $2 \times 10^6$  statistics. However the histogram of the Polyakov loop in the two sectors are in perfect agreement when one distribution is  $Z_2$  rotated as is seen clearly in Fig.3a. Apart from the Polyakov loop distributions we also compute the free energy of the different Polyakov loop sectors. In Fig.3b we show the average value of the gauge action vs  $\beta$  for the two  $Z_2$  states (called +ve and -ve) for  $N = 2$ . The gauge action for the +ve(-ve) sector is calculated by taking the average over configurations for which the Polyakov loop is +ve(-ve). The gauge actions for the two  $Z_2$  states are identical for all  $\beta$ . The free energy of each of these states can now be computed by integrating the gauge action  $S_G(\beta)$  in  $\beta$  [33, 34]. Since the gauge action are identical, the free energy will be same for the two Polyakov loop sectors.

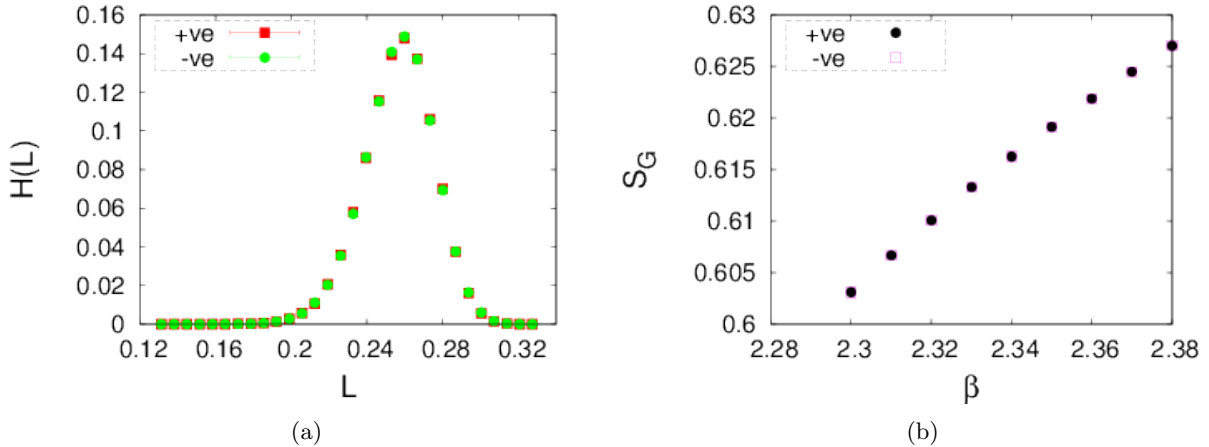


FIG. 3: (a) Comparison of Polyakov loop distributions ( $\beta = 2.38, \lambda = 0.005, \kappa = 0.056$ , lattice =  $16^3 \times 4$ ) and (b) Gauge action  $S_G(\beta)$  for the two Polyakov loop sectors for  $N = 2$ .

The confinement-deconfinement transition for  $N = 2$  for small  $\kappa$  has been investigated previously [22–26]. These studies have shown that the average value of the Polyakov loop does have critical behavior and found to be in the universality class of the Ising model. In this study for the first time we carry out the finite size scaling analysis of the Binder cumulant [35]. In Fig.4a the Binder Cumulant [35] around transition point is shown for different spatial volumes. The value of the Binder Cumulant at the crossing point corresponds to the universality class of the 3-D Ising

model. Further the scaling of the Binder Cumulant, shown in Fig.4b, gives a value for the critical exponent  $\nu$  0.62998 which is also consistent with the same universality class. These results clearly show that the confinement-deconfinement transition is second order even for finite but small  $\kappa$ . Conventionally it is thought that the confinement-deconfinement transition is true second order only for  $\kappa = 0$ . We believe that the origin of this second order confinement-deconfinement transition at  $\kappa \neq 0$  is because the fluctuations respect the  $Z_2$  symmetry. The realization of the  $Z_2$  symmetry and the critical behavior of the Polyakov loop for finite  $\kappa$  suggests that there should be a line of second order confinement-deconfinement transitions starting from  $\kappa = 0$  line on the phase diagram.

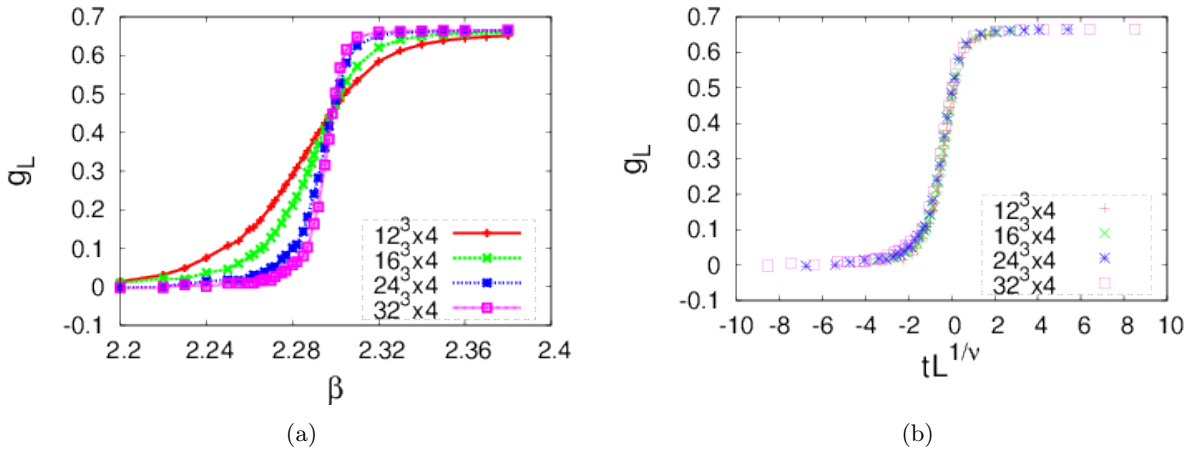


FIG. 4: (a) Binder Cumulant and (b) its scaling for  $SU(2)$ .

#### IV. DISCUSSIONS AND CONCLUSIONS

We have studied the  $Z_N$  symmetry in  $SU(N)$ +Higgs theories for  $N = 2, 3$  using numerical Monte Carlo simulations. The presence of the Higgs fields explicitly breaks the  $Z_N$  symmetry which is reflected in the asymmetry in the Polyakov loop distribution. The strength of the explicit symmetry breaking varies with the parameters  $\lambda$  and  $\kappa$ . On the other hand, given a  $(\lambda, \kappa)$  the strength does not vary much with the confinement-deconfinement transition parameter  $\beta$ . The patterns of explicit symmetry breaking observed in  $N = 2$  and  $N = 3$  are very similar. This suggests that this pattern will continue to hold for higher  $N$ .

The explicit breaking of  $Z_N$  symmetry has clear pattern along any trajectory on  $\lambda - \kappa$  plane of decreasing  $\kappa$  and the Higgs condensate. It has been observed that for large values of these variables the explicit symmetry breaking is so large that  $H(L)$  and  $H(\theta)$  have only one peak in deconfined phases. The  $Z_N$  symmetry is maximally broken in this case. Further down as  $\kappa$  and the Higgs

condensate decrease multiple peaks in the distributions do appear in the deconfined phase. For some other trajectories on the  $\lambda - \kappa$  plane, in the Higgs phase region, it is possible that only one of these two situations may arise. As the trajectory crosses the Higgs transition point  $\kappa_c$  the explicit symmetry breaking drops sharply. Close to the transition point in the Higgs symmetric phase  $H(L)$  and  $H(\theta)$  peaks are almost degenerate. It will be important to see the effect of  $N_\tau$  (number of lattice points in temporal direction) on this small but finite explicit symmetry breaking. It is possible that the explicit symmetry breaking vanishes in all of the Higgs symmetric phase in the infinite volume limit.

Conventionally it is expected that the explicit symmetry breaking will vanish only when  $\kappa$  is zero. In our simulations (with  $N_\tau = 4$ ) it is found that the explicit symmetry breaking vanishes in the Higgs symmetric phase away from the transition point. The value of  $\kappa$  for which the symmetry is restored in the theory occurs depends on  $\lambda$ . For larger  $\lambda$  the restoration of the  $Z_N$  symmetry occurs at a higher value of  $\kappa$ . This suggests that for a given  $\beta$  a line divides the  $\lambda - \kappa$  plane into  $Z_N$  symmetric and  $Z_N$  broken regions. In the  $Z_N$  symmetric region the  $Z_N$  symmetry is spontaneously broken for  $\beta > \beta_c$  which leads to  $N$  degenerate states. All physical observables such as the gauge action, the kinetic term etc. are found to be same for all the  $Z_N$  states. As a consequence the free energies of the different  $Z_N$  states are the same. Our results clearly indicate that the Higgs condensate plays the role of the  $Z_N$  symmetry breaking field. However more work is needed to relate the Higgs condensate to the effective field for the  $Z_N$  symmetry. In this work we have used the Higgs transition point to infer the values of the Higgs condensate. Since the Higgs field is not gauge invariant the Higgs condensate is not well defined. We plan to calculate the Higgs condensate by appropriately choosing a gauge which will make the Higgs condensate well defined and find out the connection between the Higgs condensate and the explicit symmetry field for  $Z_N$ .

The realization of  $Z_N$  symmetry at non-zero  $\kappa$  is in contradiction with effective potential calculations which show that the  $Z_N$  symmetry will be restored only when the Higgs mass is infinite. In these calculations only the zero mode of the Polyakov loop is coupled to the matter fields. We expect that taking care of the higher modes of the Polyakov loop will reduce the discrepancy between the non-perturbative and analytic approaches. The restoration of the  $Z_N$  symmetry in the Higgs symmetric phase has important implications for the phase diagrams of  $SU(N)$ +Higgs theories. For  $N = 2$  previously the confinement-deconfinement transition was thought to be a crossover for non-zero  $\kappa$ . Our results show that there will be a line of second order confinement-deconfinement transitions in the  $\beta - \kappa$  plane extending from the point  $(\beta_c(\kappa = 0), \kappa = 0)$ . Since the  $Z_N$  symmetry is spontaneously broken at high temperatures in the Higgs symmetric phase with vanishing

condensate it will lead to rich structures in this phase. Spontaneous symmetry breaking of the  $Z_N$  symmetry will lead to time independent topological defects solutions such as domain walls, strings etc. These defects can form even when the  $Z_N$  symmetry is mildly broken but they are not time independent and are short lived. We mention here that the restoration of  $Z_N$  symmetry may be possible in the case of gauge fields coupled to fundamental fermions as well. In this case  $\bar{\psi}\gamma_0\psi$  (which couples to the  $A_0$  field) may play the role similar to the Higgs field in restoring the  $Z_N$  symmetry.

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